CHAPTER: 6
FLOW OF WATER THROUGH SOILS

CONTENTS: Introduction, hydraulic head and water flow, Darcy's equation, laboratory determination of coefficient of permeability, field determination of coefficient of permeability, empirical formula to calculate coefficient of permeability, typical value of coefficient of permeability for different soils, flow net, one dimensional flow net, graphical representation of flow net, seepage through an earth dam on an impervious base.

6.1 INTRODUCTION
Soils are permeable due to the existence of interconnected voids through which water can flow from points of high energy to low energy. The study of the flow of water through permeable soil media is important in soil mechanics. It is necessary for estimating the quantity of underground seepage under various hydraulic conditions.

6.2 HYDRAULIC HEAD AND WATER FLOW
Consider a water flow through a soil specimen filled in a clean pipe and shown in Figure 6.1. Because of the water level difference between the left and right side of the pipe, water flows from left to right. The water level difference is called total head loss ($\Delta h$), which is the source of energy to create the flow.
Bernoulli’s equation is used to define the flow of water through soil masses in the following equation

$$Total\ head = h_x + h_p + h_v = z + \frac{u}{\gamma_w} + \frac{v^2}{2g} \ldots \ldots \ldots \ldots (6.1)$$

Where, $u$ is the pore water pressure and $v$ is the flow velocity
Head loss is an energy loss. When water flows in soils, it must flow through many small passages in void sections of soils as illustrated in Figure 6.2. This creates frictional resistance on the surfaces of particles. Flow energy is transmitted to frictional resistance on particle surfaces and then may be lost in heat generation.
6.3 Darcy’s Equation

The energy of water flow comes from the total head loss and it follows Dancy’s law in the following equation.

\[ v = ki \quad \ldots \ldots \ldots (6.2) \]

\[ q = vA = kiA = k \left( \frac{\Delta h}{L} \right) A \quad \ldots \ldots \ldots (6.3) \]

\[ Q = qt = kiAt = \left( k \frac{\Delta h At}{L} \right) \quad \ldots \ldots \ldots (6.4) \]

Where,

- \( v \): discharge velocity of water flow through porous media (m/s)
- \( k \): coefficient of permeability (m/s)
- \( i \): hydraulic gradient (head loss / flow length = \( \Delta h/L \))
- \( A \): cross sectional area of specimen perpendicular to flow direction (m\(^2\))
- \( q \): flow rate (cum/sec)
- \( Q \): total amount of flow for a period \( t \)

The discharge velocity used in the above equation is not the true velocity of water flow but is rather an average velocity in the flow direction through the porous media. Since water can flow only in the void section of the media, the true velocity must be faster than average velocity and known as seepage velocity (\( v_s \)).

\[ v_s = \frac{v}{n} \quad \ldots \ldots \ldots (6.5) \]

However, the real velocity of water molecules are even faster than true velocity since real passage are not straight but rather meandering with longer passages around the particles.

6.4 Laboratory Determination of Coefficient of Permeability

Two standard laboratory tests are used to determine the coefficient of permeability / hydraulic conductivity. These two methods are

1. Constant head test
2. Falling head test
CONSTANT HEAD TEST
The experimental setup of the constant head permeability test is illustrated in the Figure 6.3. The soil specimen is prepared in a vertical standing cylindrical mold, and a constant hydraulic head is applied. Under a steady state flow condition, discharged water at the exit is collected in a cylinder as Q for a time period t.

\[ k = \frac{QL}{A \Delta h t} \]  

Where, Q: collected amount of water for a time period t  
L: length of soil specimen in the flow direction  
A: cross sectional area of soil specimen  
\( \Delta h \): hydraulic head loss in constant head test setup  
An average value from several trials is reported as the measured k value.

FALLING HEAD PERMEABILITY TEST
The specimen is prepared similarly as in the constant head test and is shown in Figure 6.4. The higher head is applied through a burette in which the head changes with time. The head at the discharging side is constant as seen. At initial time (t=t_1), head loss is
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\[ \Delta h_1 \text{, and } t=t_2, \text{ head loss is } \Delta h_2. \text{ The amount of water flow “}q\text{” is equal to the change in head loss (}d\Delta h\text{) multiplied by the burette’s cross sectional area “}a\text{” per time “}dt\text{”.} \]

\[
q = -a \frac{d\Delta h}{dt} = \frac{k}{L} A
\]

\[
dt = \frac{aL}{Ak} \left( -\frac{d\Delta h}{\Delta h} \right)
\]

Integration from \(t_1\) to \(t_2\),

\[
\int_{t_1}^{t_2} dt = \frac{aL}{Ak} \int_{\Delta h_1}^{\Delta h_2} \left( -\frac{d\Delta h}{\Delta h} \right)
\]

\[
k = \frac{aL}{A(t_2 - t_1)} \ln \frac{\Delta h_1}{\Delta h_2} \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (6.7)
\]

The constant head is usually used for coarse grained soils and the falling head test is for finer soils.

**6.5 FIELD DETERMINATION OF COEFFICIENT OF PERMEABILITY**

The laboratory permeability test is a simple procedure. However, it should be realized that samples are reconstituted mostly for sand and gravels, and for cohesive soils,
some degree of disturbance cannot be avoided during a sampling process, transporting to the laboratory, and inserting it into the test mold. The sample size is also so small that the measured values may not be necessarily true representation of field conditions, which may include non uniformity and fissures. Thus an alternative way is to conduct field permeability test. The classic field permeability test methods involve pumping water from a well and observing water table changes in two observation wells. Two idealized field experiments are given in the subsequent sections

**UNCONFINED PERMEABLE LAYER UNDERLAIN BY IMPERVIOUS LAYER**

As seen in the Figure 6.5, a well is excavated through the permeable layer, and two observation wells are installed at \( r_1 \) and \( r_2 \) distances from the center of well hole. Water is pumped out with a steady rate until the height of water level at the well and also at the two observations wells become stable. The theory for this idealized situation gives,

\[
k = \frac{q}{\pi (h_2^2 - h_1^2)} \ln \frac{r_2}{r_1} \quad (6.8)
\]

Where, \( q \): amount of pumped water per unit time

![Figure 6.5: Field permeability test for unconfined permeable layer underlain by impervious layer](image-url)
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CONFINED AQUIFER

Figure 6.6 Shows an idealized case with a pervious layer that is sandwiched by two impervious layers. The water table is in the upper impervious layer. This situation is called confined aquifer. A well is dug into the lower impervious layer and two observations wells are installed. A steady state flow is established. The solution to compute the k value for the pervious layer in this case is

$$k = \frac{q}{2\pi H (h_2 - h_1)} \ln \frac{r_2}{r_1} \ldots \ldots \ldots \ldots \text{(6.9)}$$

Where, H: thickness of permeable layer

![Diagram of flow of water through soils](Image)

Figure 6.6: Field permeability test for confined aquifer

6.6 EMPIRICAL FORMULA TO CALCULATE COEFFICIENT OF PERMEABILITY

HAZEN'S FORMULA

Hazen's empirical formula is most widely used for saturated sandy soils.

$$K = C(D_{10})^2 \ldots \ldots \ldots \text{(6.10)}$$

Where,

$K$: coefficient of permeability (cm/s)
$D_{10}$: particle size for which 10% of the soil is finer
$C$: Hazen's empirical coefficient which ranges from 0.4 to 10.0 with average value of 1.0

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CHAPUI’S FORMULA

\[ k = 2.4622 \left( \frac{D_{10}^2}{1 + e} \right)^{0.7825} \]  \hspace{1cm} \ldots \ldots \ldots \hspace{1cm} (6.11)

KOZENY AND CARMAN’S FORMULA

A more reliable semi theoretical and semi empirical formula is given by Kozeny and Carman as

\[ k = \frac{\gamma_w}{\eta_w C_{k-c} S_s} \frac{1}{1 + e} \]  \hspace{1cm} \ldots \ldots \ldots \hspace{1cm} (6.12)

\( \eta_w \): Viscosity of water \((1.307 \times 10^{-3} \text{ N sec / sqm for } t = 10^\circ C, 1.002 \times 10^{-3} \text{ N sec/sqm for } t=20^\circ C)\)

\( C_{k-c} \): Kozeny-Carman’s empirical constant (generally 5.0 is used)

\( \gamma_w \): 9.81 KN/cum

\( S_s \): Specific surface area per unit volume of particles \((1/cm)\)

\[ S_s = \frac{SF}{D_{\text{eff}}} \]

\[ D_{\text{eff}} = \frac{100\%}{\sum f_i \cdot (D_{\text{avg},i})} \]

\[ D_{\text{avg},i} = D_{1,i}^{0.5} D_{s,i}^{0.5} \]

\( f_i \): Fraction (%) of particles between two sieve sizes with \( D_{1,i} \) as larger and \( D_{s,i} \) as smaller sieve size

SF: shape factor (spherical-6, rounded-6.1, worn-6.4, sharp-7.4, and angular-7.7)

6.7 TYPICAL VALUE OF COEFFICIENT OF PERMEABILITY FOR DIFFERENT SOILS

<table>
<thead>
<tr>
<th>Soil type</th>
<th>Coefficient of permeability (cm/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean gravel</td>
<td>100-1.0</td>
</tr>
<tr>
<td>Coarse sand</td>
<td>1.0-0.01</td>
</tr>
<tr>
<td>Fine sand</td>
<td>0.01-0.001</td>
</tr>
<tr>
<td>Silty clay</td>
<td>0.001-0.00001</td>
</tr>
<tr>
<td>clay</td>
<td>&lt;0.000001</td>
</tr>
</tbody>
</table>
6.8 FLOW NET
Flow net is a convenient graphical tool to compute hydraulic properties such as the amount of water flow, water pressure on flow boundaries, etc., for two dimensional flow problems with complex geometries. The theory of flow net can be demonstrated by using the Laplace equation for the hydraulic potential. However, a simple one dimensional model is first introduced to understand the principle of flow net.

6.9 ONE DIMENSIONAL FLOW NET
The following Figure 6.7 shows a water flow through soil in a vertical cylinder with length L and area A. the flow is downward. The cylinder is equally divided in three flow channels, which are parallel to the flow lines, and water flow never crosses the flow lines. The total head loss occurs from the top of the specimen to the bottom of the specimen as seen in the water levels in the standpipes.

![Figure 6.7: One dimensional flow net](image)

Specimen is equally divided by four blocks. Since the head loss occurs linearly through the specimen depth, the head losses between adjacent horizontal lines are all \( \Delta h/4 \).

The total heads on individual horizontal lines are constant since they have the same elevation heads and pressure heads, and thus called equipotential lines. Flow lines and equipotential lines make a net geometry, which is called flow net.
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In the flow net, equipotential lines and the flow lines intersect each other at right angles.

From figure 6.7, the amount of water flow through the small rectangular section is

\[ q_a = kia = k \frac{\Delta h}{N_d} a = k \Delta h \frac{1}{N_d} \frac{a}{b} \]

Total amount of water flow through the entire cross section is

\[ q_A = q_a N_f = k \Delta h \frac{N_f}{N_d} \frac{a}{b} \]

Taking \( a=b \)

\[ q_A = k \Delta h \frac{N_f}{N_d} \]

Here, \( \frac{N_f}{N_d} \) is called the shape factor.

6.10 GRAPHICAL REPRESENTATION OF FLOW NET

The following steps are recommended for flow net construction

1. Draw the geometry of structure correctly on the paper.
2. Select proper \( N_f \) values. Normally \( N_f \) values of 3 or 4 is adequate for the first trial.
3. Identify the boundary flow lines and boundary equipotential lines in the drawing. In an example in Figure 6.8, the upstream ground surface and downstream ground surface are the initial and the final equipotential lines, respectively. The front and backside of the sheet pile and the surface of the impervious layer are the boundary of flow lines.
4. First draw trial flow lines with selected \( N_f \) for entire earth structures. This must be done based on the engineer's best instinct on how water flows. It should be noted that there are equal amounts of water flow through all flow channels.
5. By starting from the upstream site, draw the first equipotential line to have all net openings squares or near squares with 90° intersections.
6. Continue the foregoing step for the second and third equipotential lines and so on till it reaches the downstream exit.
6.11 SEEPAGE THROUGH AN EARTH DAM ON AN IMPERVIOUS BASE

Figure 6.9 shows a homogeneous earth dam resting on an impervious base. Let the hydraulic conductivity of the compacted material of which the earth dam is made be equal to \( k \). The free surface of the water passing through the dam is given by abcd. It is assumed that a'bc is parabolic. The slope of the free surface can be assumed to be equal to the hydraulic gradient. It is also assumed that, because this hydraulic gradient is constant with depth.
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Figure 6.9: Flow through an earth dam constructed over an impervious base

\[ i \equiv \frac{dz}{dx} \]

Considering the triangle cde, we can give the rate of seepage per unit length of the dam as

\[ q = kiA \]

\[ i = \frac{dz}{dx} = \tan \alpha \]

\[ A = (ce)(1) = L \sin \alpha \]

Thus,

\[ q = KL \tan \alpha \sin \alpha \]

Again, the rate of seepage (per unit length of the dam) through the section bf is

\[ q = kiA = k \left( \frac{dz}{dx} \right) (z \times 1) = kz \frac{dz}{dx} \]

For continuous flow,

\[ kz \frac{dz}{dx} = KL \tan \alpha \sin \alpha \]

\[ \int_{z=H}^{z=L \sin \alpha} kzdz = \int_{z=0}^{z=L \cos \alpha} KL \tan \alpha \sin \alpha \, dx \]

Solving the above integration,

\[ L = \frac{d}{\cos \alpha} - \sqrt{\frac{d^2}{\cos^2 \alpha} - \frac{H^2}{\sin^2 \alpha}} \]
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Following is a step-by-step procedure to obtain the seepage rate \( q \):

1. Obtain \( \alpha \).
2. Calculate \( \Delta \) and then \( 0.3 \Delta \)
3. Calculate \( d \).
4. With known value of \( \alpha \) and \( d \), calculate \( L \)
5. With known value of \( L \), calculate \( q \)

[TRY TO SOLVE]

[1] In a constant head permeater test, the following observations were taken.

- Distance between piezometer tappings = 100mm
- Difference of water levels in piezometers = 60mm
- Diameter of the test sample = 100mm
- Quantity of water collected = 350ml
- Duration of the test = 270 sec

Determine the coefficient of permeability.

[2] The falling head permeability test was conducted on a soil sample of 4cm diameter and 18cm length. The head fell from 1.0m to 0.40m in 20 minutes. If the cross sectional area of the standpipe was 1 cm\(^2\), determine the coefficient of permeability.

[3] A sandy layer 10m thick overlies an impervious stratum. The water table is in the sandy layer at a depth of 1.5m below the ground surface. Water is pumped out from a well at the rate of 100 liters per second and the drawdown of the water table at radial distances of 3.0m and 25.0 m is 3.0 and 0.50m, respectively. Determine the coefficient of permeability.

[4] A layered soil is shown in the following figure. Given that,

- \( H_1 = 1 \text{m} \), \( k_1 = 10^{-4} \text{ cm/sec} \)
- \( H_2 = 1.5 \text{m} \), \( k_2 = 3.2 \times 10^{-2} \text{ cm/sec} \)
- \( H_3 = 2 \text{m} \), \( k_3 = 4.1 \times 10^{-5} \text{ cm/sec} \)

Estimate the ratio of equivalent permeability
Problem - 4

[5] Flow net under a concrete dam is drawn in following figure.

(a) Calculate and plot the water pressure distribution along the base of the dam.

(b) Compute the resultant uplift force against the base of the dam.

(c) Calculate the point of application of the resultant uplift force.
[6] An earth dam is shown in the following figure. Determine the seepage rate, $q$ in $m^3/day/m$ length. Given, $\alpha_1=\alpha_2=45^0$, $L_1=5$ m, $H=10$ m, $H_1=13$ m, and $k=2\times10^{-4}$ cm/sec.